

$$\frac{2}{3} \leq AP^2 + AQ^2 \leq \frac{8 - 2\sqrt{6}}{3}$$